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HIGH VOLTAGE BIREFRINGENCE MEASUREMENTS OF ELASTIC CONSTANTS

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ABSTRACT

It is shown that the birefringence change associated with an electric field-induced planar to homeotropic Freedericksz transition can be linearly related to the applied voltage for large director deformations. Values for the bend:splay elastic constant ratio can be obtained from a linear fit of voltage to inverse birefringence. Experimental results are presented for 5CB which are in reasonable agreement with values obtained by other techniques.

INTRODUCTION

Torsional elasticity is the most characteristic property of nematic liquid crystals, and the principal elastic constants can be related to the anisotropic intermolecular potential responsible for the formation of the orientationally ordered phase.[1,2] The same elastic constants are parameters of the Frank-Oseen continuum theory of nematics, and determine the response of liquid crystals to external fields and hence the operation of liquid crystal display devices. Unfortunately the measurement of torsional elastic constants is a delicate matter and great care is necessary to obtain reliable results. Two main methods have been adopted to measure elastic constants: one uses the analysis of Freedericksz transitions[3], while the other relies on the information contained in the light scattered by director fluctuations.[4]

Elastic constants may be obtained from electric/magnetic Freedericksz transitions by measuring the threshold voltage/

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field or by fitting the optical or capacitative response of the cell above threshold to the continuum equations.[5,6] There are difficulties associated with both these approaches: threshold fields for bend and twist elastic constants are difficult to determine precisely, while numerical fitting of the cell response is a very sensitive procedure and may introduce unsuspected systematic errors.[7] For small field-induced distortions of the director it is possible to expand the continuum equations to give a linear dependence of the birefringence or capacitance change on applied field strength[8], where the slope is a function of the elastic constants and susceptibility (electric or magnetic) anisotropy. However, the range over which the linear approximation is valid is very limited[8], and results using the linear approximation can be misleading.[9]

It is possible to simplify the continuum equations for high field strengths[10], and this has been proposed[11] as a new method for determining elastic constants from capacitance/voltage curves. In this letter we derive the high field continuum equations that describe the change in birefringence of an initially planar aligned positive nematic subjected to an electric field. Experimental results are reported for the high field birefringence of 44'n-pentylcyanobiphenyl (5CB), and the results are analysed to provide new values for the elastic constants of this material.

THEORY

The equations relating the change in birefringence (δ) of a planar cell to applied voltage (V) were given by Gruler, Scheffer and Meier[5] and Deuling[6]:

$$\left(1 - \frac{\delta\lambda}{n_e \ell}\right)\int_0^{\phi_m} \frac{(1 + \kappa \sin^2\phi)^{1/2}(1 + \gamma \sin^2\phi)^{1/2}}{(\sin^2\phi_m - \sin^2\phi)^{1/2}}\, d\phi =$$

$$\int_0^{\phi_m} \frac{(1 + \kappa \sin^2\phi)^{1/2}(1 + \gamma \sin^2\phi)^{1/2}}{(1 + \nu \sin^2\phi)^{1/2}(\sin^2\phi_m - \sin^2\phi)^{1/2}}\, d\phi \qquad (1)$$

and

$$\frac{V}{V_o} = \frac{2}{\pi}(1 + \gamma \sin^2\phi_m)^{1/2}\int_0^{\phi_m} \frac{(1 + \kappa \sin^2\phi)^{1/2}}{(1 + \gamma \sin^2\phi)^{1/2}(\sin^2\phi_m - \sin^2\phi)}\, d\phi \qquad (2)$$

where ϕ_m is the maximum deformation angle of the director in the centre of the cell, V_o is the threshold voltage, ℓ is the cell thickness and λ is the wavelength of light used to measure the birefringence. The material parameters that enter these equations are $\gamma = (\varepsilon_{\parallel} - \varepsilon_{\perp})/\varepsilon_{\perp}$, $\kappa = (k_{33} - k_{11})/k_{11}$ and $\nu = (n_e^2 - n_o^2)/n_o^2$, where $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ are the components of the permittivity tensor, n_e and n_o are the extraordinary and ordinary refractive indices and k_{33} and k_{11} are the bend and splay elastic constants. The birefringence change δ is

$$\delta = \frac{1}{\lambda}\int_o^{\ell} [n_e - n(z)]dz \qquad (3)$$

where n(z) is the refractive index for light polarised in the plane of the director and the electric field at a distance z inside the liquid crystal film.

In the limit of high electric fields ϕ_m tends to $\pi/2$, but the integrals in eqs. (1) and (2) diverge at this limit. Making the following substitution:

$$\delta = d - \frac{\ell n_o}{\lambda}[1 - (\nu + 1)^{1/2}] \qquad (4)$$

it is possible to rearrange eq. (1) to give:

$$\frac{\lambda d}{n_o \ell}\int_o^{\phi_m} \frac{(1 + \kappa \sin^2\phi)^{1/2}(1 + \gamma \sin^2\phi)^{1/2}}{(\sin^2\phi_m - \sin^2\phi)^{1/2}}\, d\phi =$$

$$(5)$$

$$\int_o^{\phi_m} \frac{(1 + \kappa \sin^2\phi)^{1/2}(1 + \gamma \sin^2\phi)^{1/2}}{(\sin^2\phi_m - \sin^2\phi)^{1/2}} \left[1 - \left(\frac{1 + \nu}{1 + \nu \sin^2\phi}\right)^{1/2}\right] d\phi$$

neither side of which diverges in the high field limit. Further rearrangement of the L.H.S. of eq. (5) gives:

$$(1 + \gamma)\int_o^{\phi_m} \frac{(1 + \kappa\sin^2\phi)^{1/2}}{(\sin^2\phi_m - \sin^2\phi)^{1/2}(1 + \gamma\sin^2\phi)^{1/2}}\, d\phi$$

$$- \gamma\int_o^{\phi_m} \frac{(1 + \kappa\sin^2\phi)^{1/2}\cos^2\phi}{(\sin^2\phi_m - \sin^2\phi)^{1/2}(1 + \gamma\sin^2\phi)^{1/2}}\, d\phi =$$

$$= \frac{n_o \ell}{\lambda d} \int_0^{\phi_m} \frac{(1 + \kappa \sin^2\phi)^{1/2}(1 + \gamma \sin^2\phi)^{1/2}}{(\sin^2\phi_m - \sin^2\phi)^{1/2}} \left[1 - \left(\frac{1 + \nu}{1 + \nu \sin^2\phi}\right)^{1/2}\right] d \qquad (6)$$

and using eq. (1) we obtain:

$$\frac{\pi V(1 + \gamma)}{2V_o(1 + \gamma \sin^2\phi_m)^{1/2}} - \gamma A(\phi_m) = \frac{n_o \ell}{\lambda d} B(\phi_m) \qquad (7)$$

The functions $A(\Phi_m)$ and $B(\Phi_m)$ are defined by eqs. (6) and (7), and in the limit of $\Phi_m = \pi/2$, they become:

$$A = \int_0^1 \frac{(1 + \kappa x^2)^{1/2}}{(1 + \gamma x^2)^{1/2}} dx \qquad (8)$$

$$B = \int_0^1 \frac{(1 + \kappa x^2)^{1/2}(1 + \gamma x^2)^{1/2}}{(1 - x^2)} \left[1 - \frac{(1 + \nu)^{1/2}}{(1 + \nu x^2)^{1/2}}\right] dx \qquad (9)$$

where $x = \sin \phi$

Thus at high voltages eq. (7) may be written as:

$$\frac{V}{V_o} = \frac{2 n_o \ell B}{d \pi \lambda (1 + \gamma)^{1/2}} + \frac{2 \gamma A}{\pi (1 + \gamma)^{1/2}} \qquad (10)$$

and the voltage is a linear function of the inverse of the birefringence (d). Unfortunately the material constants A and B cannot be expressed as elementary functions of κ, γ and ν, but comparison of experimental and calculated results for A and B will yield two of the parameters κ, γ and ν. Usually both ν and γ are known independently and so experimental values for A and B will provide two independent measurements of κ.

RESULTS AND DISCUSSION

The change in birefringence during the Freedericksz transition was measured by placing the planar cell between crossed polariserswith the initial director axis at 45° to the polarisation axis. The intensity of transmitted light (λ = 633nm) was measured as a function of applied voltage, and from these measurements and knowing the thickness of the liquid crystal film, the birefringence change could be calculated. Increased precision was achieved by chopping the incident light and using a phase-sensitive detector. The temperature of the cell was controlled to better than ±0.1°C.

A typical plot of light intensity against voltage in the high field region is shown in figure 1, and in figure 2 plots of V/V_0 against d^{-1} are given for three temperatures. The continuous lines in figure 2 are calculated using eq. (10) and the physical parameters listed in table 1. Values of V_0 ν and γ have been determined in separate experiments, ℓ was measured to be 30.8μm, and the κ values correspond to a best fit to the experimental points. Also listed in the table are measurements of elastic constants for 5CB reported by other workers.

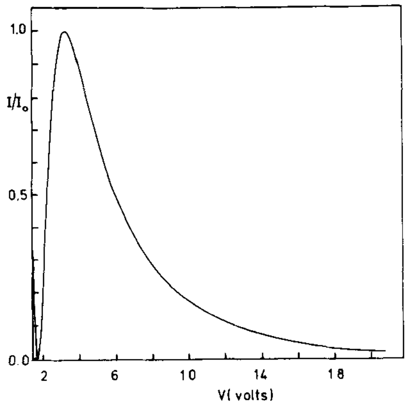

FIGURE 1: Intensity plot against voltage

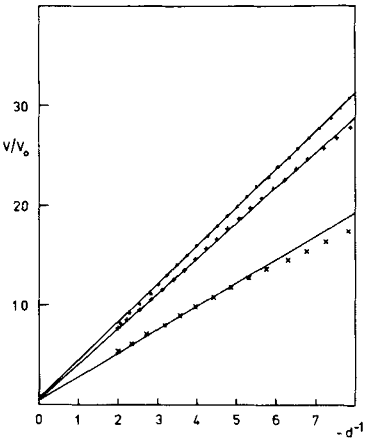

FIGURE 2: V/V_0 against $-d^{-1}$

TABLE 1 - Physical Properties of 5CB

temp. (°C)	ν	γ	κ	V_0 (volts)	k^{11} (10^{-12}N)	k_{33} (10^{-12}N)
28.5	0.2276	1.6143	0.232	0.698	4.94^a 5.30^b 5.65^c	6.1^a 6.8^b 7.2^c
31.5	0.2093	1.3946	0.093	0.665	4.07^a 4.30^b 4.60^c	4.5^a 5.3^b 5.6^c
34.6	0.1595	1.0563	-0.8	0.600	2.74^a 2.80^c	0.6^a 3.2^c

a - this work; b - ref. (12); c - ref (13).

Our measurements away from the nematic to isotropic transition are in reasonable agreement with results quoted by some other workers and the relatively small differences noted can be partly attributed to the use of different values of γ. Close to the transition (T_{NI} = 35.2°C) we observed a divergence from the linear response predicted by theory, and the fitted bend elastic constant is very small. This behaviour is unexpected, and may be attributed to either pretransitional effects or a breakdown in the strong anchoring assumption. Both coefficients A and B in eq. (10) contain information on the elastic constants and are approximately linearly dependent on κ, but in practice the intercept A is always small and subject to considerable error. One advantage of high field birefringence measurements over the corresponding capacitance measurements is that the former can be made with greater precision over a limited range of high voltages; it is only in this region that the linear approximation can be used with confidence. For example our measurements encompassed a relative birefringence change of less than 14%. The determination of elastic constants using high field measurements still requires some numerical analysis, but it is not as sensitive as fitting the response curve above threshold. Although the technique is not a replacement for existing methods, it does provide an independent check on often discordant results.

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